

Local galactic disk stability under MONDian gravity

M. A. Jiménez and X. Hernandez

*Instituto de Astronomía, Universidad Nacional Autónoma de México, Apartado Postal 70–264 C.P. 04510
México D.F. México*

ABSTRACT

Toomre’s Q stability parameter has long been shown through various theoretical arguments and numerical simulations, to be the principal determinant of stability against self-gravity in a galactic disk, under classical gravity. Comparison with observations however, has not always confirmed the condition of $Q = Q_{crit}$ to be well correlated with various critical galactic radii. In this paper we derive the analogous critical parameter, Q_M , under MONDian gravity. The result is a modification by a factor of $(\sigma\Omega)/a_0$, $Q_M = (\sigma\Omega)^2/(a_0G\Sigma)$, where a_0 is the critical acceleration scale of MOND. We then show through a direct comparison to a homogeneous sample of observed disk galaxies with measured brightness profiles, rotation velocity curves and internal velocity dispersion profiles, that the critical radii at which brightness profiles dip below the exponential fit, are much more accurately predicted by Q_M than by the Q of classical gravity. This provides a new and completely independent argument supporting the reality of a change in the form of gravity on reaching the low acceleration regime.

Subject headings: gravitation — galaxies: general — galaxies: kinematics and dynamics — galaxies: star formation

1. Introduction

Toomre’s Q parameter (Toomre 1964) neatly encompasses a comparison of the disruptive total tidal forces and internal dynamical pressure, against local self gravity, for a local element of a galactic disk. It remains the most basic measure of the stability of a galactic disk, a valuable diagnostic in evaluating the propensity of a disk towards spiral arm, bar or bulge formation e.g. Athanassoula (2003). Also, Q has often been related to star formation processes in spiral galaxies. The idea being that in very hot disk regions, above certain critical Q values, the self gravity of a gaseous element is insufficient to overcome the combined effects of internal pressure and tidal shears, and is hence incapable of initiating collapse and undergoing star formation. The expectation then arises for surface brightness profiles to show truncations or kinks at the critical radii where Q crosses some threshold value.

In terms of an evolutionary view of a galactic disk, all the parameters upon which Q depend, the rotation curve, the surface density profile and the internal velocity dispersion one, are expected to change over the course of a disk galaxy’s growth and lifetime e.g. Firmani et al. (1996). Thus, the present day surface brightness profile, an integral over the galactic lifetime of the star formation processes, will retain only a somewhat averaged out version of the history of this critical radius and its evolution e.g. Schaye (2004). However, observational studies seeking to relate features in the surface density profiles of disks to estimates of Q , have often found only weak correlations with critical Q values e.g. the subcritical large Q star-forming disks of Martin & Kennicutt (2001).

In view of the above, we find it interesting to develop the analogous stability parameter under MONDian gravity, Q_M , and to perform a first observational test of this new parameter. MONDian gravity, where the force felt by a test particle orbiting a mass M changes from the Newtonian value to $(a_0GM)^{1/2}/R$, was originally presented

²email:xavier@astro.unam.mx

as a way of understanding the observed asymptotic flatness of galactic rotation curves in the absence of any dark matter, Milgrom (1983). Since, modified gravity schemes that reduce to the above MONDian behaviour, have been shown to be accurately in accordance with a range of gravitational astrophysical observations and scales; in Hernandez et al. (2012) some of us showed solar neighbourhood wide binary stars to have relative velocities which deviate from Keplerian expectation, precisely on crossing the a_0 threshold, Haghi et al. (2011), Scarpa et al. (2011) and Hernandez et al. (2013a) showed that the same transition is observed in Galactic globular clusters, which show consistency also with a galactic Tully-Fisher relation, Kroupa (2012) has given an accurate description of the dynamics and orbits of dwarf galaxies, and extending the well known description of Spiral rotation curves under MOND (e.g. Sanders & McGaugh 2002, Swaters et al. 2010), in Jimenez et al. (2013) some of us presented a full description of the massive elliptical galaxy NGC 4649, all of the above cases, without the substantial fractions of dark matter which a classical description requires.

In the above, the range of gravitational phenomena studied under MONDian gravity has been progressively extended from equilibrium rotation velocities, to an ever growing range of aspects. This provides increasingly more independent empirical tests, e.g. the isothermal distributions of MONDian gravity which accurately and naturally reproduce the observed $\rho \propto r^{-3}$ density profiles of tenuous stellar halos surrounding external galaxies, Hernandez et al. (2013b). In this paper we continue along the same lines, by developing a MONDian analogue to the standard first order Toomre’s stability criterion for disk galaxies, and preform a first test using the recent and homogeneous galactic structure atlas of Martinsson et al. (2013a) and Martinsson et al. (2013b).

Global disk stability under MOND has been treated by Milgrom (1989), however, within the limits of tightly wound global, purely surface density perturbations, imposing an infinitely thin disk. This last constraint, one of treating an essentially 3D problem within a 2D approximation, leads to the modified Toomre criterion of Milgrom (1989) where only an order unity correction with respect to the classic case appears through the di-

mensionless MOND transition function, and where the critical acceleration of a_0 does not appear.

Section (2) presents a first order local 3D derivation of Toomre’s Q stability parameter under MONDian gravity, Q_M . In section (3) we use a sample of disk galaxies from Martinsson et al. (2013a) and Martinsson et al. (2013b) with measured surface density light profiles, rotation velocities and internal disk dispersion velocities, to test the relevance of Q_M in terms of predicting critical radii at which the surface brightness profile falls below the exponential fit. We see from the data that indeed, Q_M offers a much better description of the observed break radii than Q . Our conclusions are presented in section(4).

2. Theoretical expectations

In the study of classical gravitational instabilities in galactic disks, the Toomre stability criterion is regarded as the most valuable diagnostic. This is defined as:

$$Q_T = \frac{\sigma \kappa}{\alpha G \Sigma}, \quad (1)$$

In the above, $\kappa(R)$ is the epicyclic frequency of the disc, related to the orbital frequency in the disk, $\Omega(R)$, through $\kappa^2(R) = R d\Omega^2/dR + 4\Omega^2$, $\sigma(R)$ represents the velocity dispersion of disk material, $\Sigma(R)$ the stellar surface density and α is a numerical constant of order one which varies slightly for the cases of non-collisional stellar components, dissipative gaseous disks, mixed components, the presence of magnetised gas, etc. Binney & Tremaine (1987). The above stability criterion arises from the dispersion relation for infinitely thin disks (Toomre 1969), but has been shown through extensive numerical simulations to be the most relevant first order stability determinant for realistic galactic disks, when assuming standard gravity.

Under Newtonian gravity one can understand the basic physics involved in Toomre’s stability criterion by considering a parcel of disk material of density ρ moving at the orbital frequency of the disk at radius R , $\Omega(R)$. Such a parcel will be prevented from collapsing onto itself due to its self gravity by the overall tidal forces, if its density is roughly below that of the total matter density interior to R , $\rho < \bar{\rho}(R)$, i.e. it fails to satisfy a tidal

density criterion. To within a numerical constant of order one, $\Omega^2(R) = G\bar{\rho}(R)$, so we can write the tidal stability criterion:

$$\rho < \frac{\Omega^2}{G} \quad (2)$$

Taking $\Sigma = \rho h$, with h the typical scale height of the disk, yields

$$h_t > \frac{G\Sigma}{\Omega^2} \quad (3)$$

as the critical disk scale above which tides will stabilise local self-gravity.

On the other hand, internal pressure will stabilise the disk element against its self-gravity, below a critical Jeans scale, $h_J = \sigma/(G\rho)^{1/2}$. Taking again $\Sigma = \rho h$ gives:

$$h_J < \frac{\sigma^2}{G\Sigma} \quad (4)$$

as the critical disk scale below which internal pressure will stabilise local self-gravity.

The disk will be locally stable against perturbations of all scales below a critical Σ such that $h_J = h_t$, as below this critical Σ , $h_J > h_t$, with the equalling of the two critical scales above giving the condition:

$$Q = \frac{\sigma\Omega}{G\Sigma} = 1 \quad (5)$$

as the critical criterion. For $Q > 1$ the disk is locally stable against perturbations of all scales. In eq.(5) above, we see the numerator representing dynamical pressure and tides through σ and Ω as stabilising factors against the local self-gravity in the denominator through $G\Sigma$.

We see that this last expression neatly captures the essential physics behind $Q_T = 1$ of eq.(1) being the relevant disk stability criterion, specially considering that this critical radius is expected to occur within the flat rotation curve region of a galactic disk, where $\kappa = 2^{1/2}\Omega$. For a canonical value of $\alpha = \pi$ in eq.(1), the critical point would appear as $(0.45\sigma\Omega/G\Sigma) = 1$ within the flat rotation curve region. The above development is well known (e.g. Binney & Tremaine 1987), but is repeated here to make the analogy with the development under modified gravity which follows more explicit.

In going to a MONDian modified gravity regime, for accelerations below a critical value of a_0 , the force felt by a test particle orbiting a spherically symmetric mass distribution of total mass M becomes $F_M = (a_0GM)^{1/2}/R$, and hence rotation velocities become flat at a value consistent with the observed Tully-Fisher relation, $V_f = (a_0GM)^{1/4}$. It is easy to see that the resulting tidal criterion for a spherically symmetric mass element now becomes $\rho < (h_{tM}/R)\bar{\rho}(R)$ e.g. Hernandez & Jimenez (2012). Since the critical point will occur at radii such that the integrated mass of the galactic disc has essentially converged, $V_f^4 = a_0G\bar{\rho}(R)R^3$. Using again $\Sigma = \rho h$ to eliminate ρ in favour of the Σ of eq.(5), the critical scale above which tides will stabilise local self-gravity becomes:

$$h_{tM} > \frac{(a_0G\Sigma)^{1/2}}{\Omega^2} \quad (6)$$

The modified version of the Jeans scale, the balance between internal dynamical pressure and self-gravity is now $m_{JM} = \sigma^4/Ga_0$, with parcels of mass below m_{JM} being stabilised by their internal dynamical pressure, e.g. Mendoza et al. (2011). Taking $m_{JM} = \Sigma h_{JM}^2$ to replace m_{JM} for the Σ of eq.(5) leads to the condition:

$$h_{JM} < \frac{\sigma^2}{(a_0G\Sigma)^{1/2}} \quad (7)$$

Scales below h_{JM} will be stabilised by internal velocity dispersion. Following the analogy with the Newtonian derivation of eq.(5), we see that under the modified regime, the disk will be locally stable against perturbations of all scales below a critical Σ such that $h_{JM} = h_{tM}$, as below this critical Σ , $h_{JM} > h_{tM}$, with the equalling of the two critical scales above giving the condition:

$$Q_M = \frac{(\sigma\Omega)^2}{a_0G\Sigma} = 1 \quad (8)$$

as the critical criterion under modified gravity. For $Q_M > 1$ the disk is locally stable against perturbations of all scales. Introducing the characteristic surface density of MOND, $\Sigma_M = a_0/G$, we can write the critical surface density which yields $Q_M = 1$ as:

$$\Sigma_c = \Sigma_M \left(\frac{\sigma\Omega}{a_0} \right)^2 \quad (9)$$

We see that the result of eq.(8) introduces a dimensionless factor of $(\sigma\Omega/a_0)$ to the classical expression of eq.(5) i.e., $Q_M = (\sigma\Omega/a_0)Q$. This correction factor will be large when the internal disk acceleration scale is much smaller than a_0 . Thus, we see that a galactic disk which might appear as “sub-critical” in terms of surface density because its $Q > 1$, could quite easily have a low $Q_M < 1$ ($\sigma\Omega/a_0 < 1$) and hence be naturally understood as hosting the observed levels of star formation. For values of $\Sigma > \Sigma_M$, the classical Q would apply.

It is interesting that under classical gravity stability to both global tight winding patterns under a 2D treatment, and a local bulk self-gravity vs. dynamical pressure and tides balance give essentially the same result of Q , while for MONDian gravity, the former leads to the modified Q of Milgrom (1989) where only a dimensionless correction factor given by the MOND transition function appears, and the latter to the Q_M of eq.(8). Notice also that stellar disk critical radii occur in the flat rotation curve region, and at surface densities well below the critical MOND value of a_0/G (e.g, Famaey & McGaugh 2012), making the preceding development valid not only for a MONDian gravity scheme where no external field effect is included, but also under MOND as such.

We can reach a slightly more approximate, but more easily testable prediction by taking the following empirically grounded models for $\Sigma(R)$ and $\sigma(R)$:

$$\Sigma(R) = \Sigma_0 e^{-(R/R_\star)} \quad (10)$$

$$\sigma(R) = \sigma_0 e^{-(R/R_\sigma)} \quad (11)$$

Taking the common observational result of $R_\sigma \approx 2R_\star$, if we now evaluate Q_M at R_c , the critical galactic radius, where $Q_M = 1$, we obtain the relation:

$$\frac{\sigma_0^2 \Omega^2(R_c)}{a_0 G \Sigma_0} = 1 \quad (12)$$

Multiplying above and below by R_\star^2 , and using the relation $M = 2\pi R_\star^2 \Sigma_0$ for galactic discs where

again $a_0 G M = V_f^4$, with V_f is the flat rotation curve disk orbital velocity yields,

$$\frac{R_c}{R_\star} = \beta \frac{\sigma_0}{V_f}, \quad (13)$$

where β is a proportionality constant to account for the various first order estimates introduced into the previous development. Thus, we reach an easily testable prediction for the critical disk radii, in units of the stellar disk scale length, to be proportional to the quotient of the central disk dispersion velocity to the flat rotation velocity value, if MONDian dynamics are important to real galactic disks.

3. Comparisons with real galaxies

We begin with a first test using only eq.(13), where use of eq.(10) and eq.(11) allows to eliminate the surface mass density of the disk, a parameter which is subject to significant uncertainties due to the large error intervals in mass to light ratios, gas fractions, etc. The galactic parameters needed for testing the relevance of the stability criteria for galactic disks under MONDian gravity of eq.(13) are: the value of the rotation velocity within the flat rotation curve region V_f , the σ_0 parameter from a full disk fit of eq. (11) to the velocity dispersion profile, and R_\star and R_c , the disk scale, and the critical galactic radius where the brightness surface profile dips below the exponential fit. Notice that σ_0 is not the actual $\sigma(R=0)$, which would mostly reflect central bulge dynamics, and not the velocity dispersion representative of the disk at large radii through eq.(11).

Fortunately, all these parameters appear in the DiskMass Survey of Martinsson et al. (2013a), which consists of a sample of thirty disk galaxies, with their corresponding rotation curves, the velocity dispersion profiles of the disk stars measured along the line of sight and perpendicular to the disk plane, the K- band surface brightness profiles, and observed disk scale lengths for each galaxy in the sample. We prefer to work with a homogeneous sample where all the quantities for all the galaxies have been measured and treated consistently by the same authors, rather than attempt to use a larger heterogeneous sample where any trend will always be suspect to having arisen through the shifting systematics across the sam-

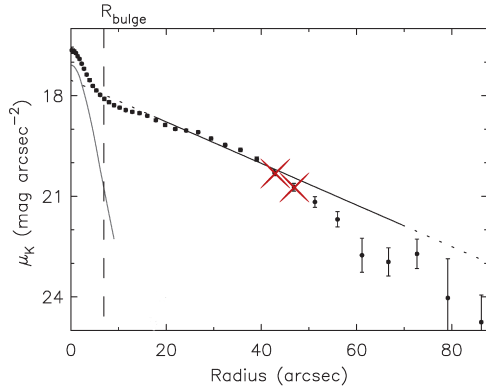


Fig. 1.— The figure shows how we take the critical radius, R_c of each galaxy, from their K-band surface brightness profiles, here is shown an example for *UGC4036*, the profile falls below the exponential fit between the points marked with crosses the midpoint is $R_c = 44.8 \text{ arcsec}$ the critical radius for the galaxy.

ple.

To test eq.(13) we use observational the quantities V_f and R_* from Martinsson et al.(2013a) and Martinsson et al.(2013b), R_c is obtained directly from K-band surface brightness profile, we take σ_0 from the exponential fits to the velocity dispersion profile presented in the same work. In fact we use the fit for σ_z and use the parameter $\sigma_{z,0}$ which is not affected by the varying degrees of inclination at which the different galaxies are observed. We assigned an R_c value to each galaxy as the radius where its brightness profile drops below the exponential fit.

As an example of how we take the critical radius R_c in each galaxy we reproduce the K-band surface brightness profile for galaxy *UGC4036* as it appears in Martinsson et al.(2013a) in figure (1), from the figure we can see that the profile falls below the exponential fit between the points marked with crosses, we take the midpoint $R_c = 44.8 \text{ arcsec}$ as the critical radius for *UGC4036*, The error associated to R_c is the radial interval between the points where the fall occurs. We proceed in the same way for all galaxies and use only those where the drop is evident, which leaves us with a subsample of twenty disk galaxies.

In figure (2) we plot $\frac{R_c}{R_*}$ and $\frac{\sigma_0}{V_f}$ for the galax-

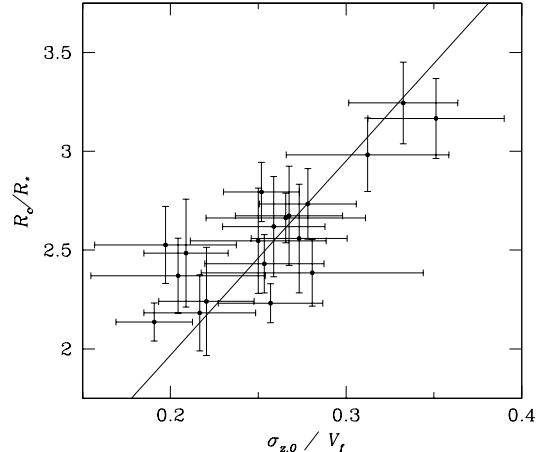


Fig. 2.— The figure shows the critical radius where the light profile of each galaxy drops below the exponential fit, R_c , in units of the disk light disk scale length, R_* , of each system, as a function of the ratio of the central disk vertical velocity dispersion, $\sigma_{z,0}$ to the flat rotation velocity amplitude of each galaxy. The solid line gives the best fit direct proportionality between the two quantities, a result expected from the MONDian disk stability analogue to the classic Toomre stability criteria, and which can be seen to be an excellent fit to the observational data.

ies in our sample. It can be seen that the result is consistent with a straight line, as predicted by eq.(13), with $\beta = 9.84 \pm 0.24$. After assigning an error to R_c as described above, we perform the full error propagation to calculate the corresponding 1σ confidence intervals for the quantities $\frac{R_c}{R_*}$ and $\frac{\sigma_0}{V_f}$ using the errors reported in Martinsson et al.(2013a) for V_f , R_* and $\sigma_{z,0}$ to arrive at the error bars show in the figure.

We note that for our sample $\langle \frac{R_*}{R_{\sigma z}} \rangle = 0.43$ with a dispersion of 0.21, supporting the assumption used to derive the eq. (13) of $R_\sigma \approx 2R_*$. *UGC4455* was not considered for our study, as it is the only galaxy in Martinsson et al. (2013a) described by the authors as having strong spiral arms which might disrupt the determination of the surface brightness profile. Besides from *UGC4455*, no

galaxy with a clearly evident dip below the exponential profile was excluded from our sample, the 10 excluded ones show no evident dip at the radii predicted by eq.(13), or at any other one. This probably reflects the sensitivity of K-band observations to the integral star formation history of a galaxy, if changes in surface density, stellar and gas mass profiles, and velocity dispersion profiles (or amplitude of the rotation velocity curve even) have shifted R_c significantly during the course of evolution, no trace of this feature would be imprinted upon the present K-band surface brightness profile. A better test of the ideas here presented would require a large homogeneous sample including an indication of the ionisation state of the gas along the disk.

A more detailed calibration of the numerical proportionality factor at the critical radii can be obtained from a further subset of galaxies where all the relevant quantities are directly available at R_c , and no global fits are used. For these we obtain $\langle(\sigma/\Omega)^2/(a_0 G \Sigma)\rangle = 0.4$ at the critical radii. Notice that this last number can not be directly compared to the previous, more approximate calibration, of $\beta = 9.84$, as passing from eq.(8) to eq.(13) implies assuming $M = 2\pi R_\star^2 \Sigma_0$, where M is the total baryonic mass of the galaxy, something that is not exactly accurate, to a degree which depends on the bulge fraction, gas fraction, and details of the mass to light ratios (and their probable radial variations) used in estimating the surface density mass profile from the observed surface brightness one. Notice also that if one is to assess the validity of a modified gravity law, Σ estimates which rely on classical gravity virial relations for the vertical disk structure are not relevant. Using the same data above to evaluate the classical $\langle Q \rangle = \langle(\sigma/\Omega)/(G\Sigma)\rangle$ at the critical radii gives a value of $\langle Q \rangle = 4$, and hence the disks studied here appear classically as the low surface density 'sub-critical' hot, large Q systems of Martin & Kennicutt (2001).

We end this section with Table (1), which summarises the galaxies and parameters used in this study.

4. Conclusions

We develop under MONDian gravity the equivalent to Toomre's first order disk stability crite-

riion, by analogy to the classical case, through a comparison of the tidal critical density and a Jeans critical density. Our result is hence a local criterion referring to the ability of a 3D disk element to undergo collapse under its self-gravity.

The resulting critical parameter can be found to be $Q_M = (\sigma\Omega/a_0)Q = (\sigma\Omega)^2/(a_0 G \Sigma)$.

A comparison to the critical radii at which the observed surface light profile drops below the exponential fit, in a sample of disk galaxies with measured rotation curves and velocity dispersion profiles, shows much better agreement with a Q_M criterion than with a classic Q one.

It appears that the physics of tides and internal dynamical pressure balancing self-gravity in disk galaxies is much better represented by MONDian physics than by classical gravity.

5. Acknowledgements

Xavier Hernandez acknowledges financial assistance from UNAM DGAPA grant IN100814. Alejandra Jimenéz acknowledges financial support from a CONACYT scholarship.

REFERENCES

- Binney J., Tremaine S., 1987, *Galactic Dynamics* (Princeton University Press, Princeton, NJ)
- Famaey B., McGaugh S. S., 2012, *LRR*, 15, 10
- Firmani C., Hernandez X., Gallagher J., 1996, *A&A*, 308, 403
- Haghi H., Baumgardt H., Kroupa P., 2011, *A&A*, 527, A33
- Hernandez X., Jimenez M. A., Allen C., 2012, *EPJC*, 72, 1884
- Hernandez X., Jimenez M. A., Allen C., 2013a, *MNRAS*, 428, 3196
- Hernandez X., Jimenez M. A., Allen C., 2013b, *ApJ*, 770, 83
- Jimenez M. A., Garcia G., Hernandez X., Nasser L., 2013, *ApJ*, 768, 142
- Kroupa P., 2012, *PASA*, 29, 395
- Athanassoula E., 2003, *MNRAS*, 341, 1179

Table 1: Parameters for the galaxies treated.

UGC	$V_f(km/s)$	$R_*(kpc)$	$\sigma_{z,0}(km/s)$	$R_c(kpc)$
448	186.00 ± 11.00	3.86 ± 0.21	47.8 ± 2.70	8.61 ± 0.47
463	209.00 ± 12.00	3.78 ± 0.27	69.5 ± 2.50	12.26 ± 0.77
1081	156.00 ± 9.00	3.05 ± 0.28	43.40 ± 1.80	8.35 ± 0.62
1087	160.00 ± 1.00	3.23 ± 0.21	42.50 ± 4.60	8.61 ± 0.45
1635	152.00 ± 9.00	2.92 ± 0.21	29.00 ± 1.60	6.24 ± 0.42
1862	102.00 ± 8.00	1.40 ± 0.39	26.40 ± 0.90	3.68 ± 0.51
1908	237.00 ± 14.00	4.86 ± 0.19	74.00 ± 6.60	14.51 ± 0.70
3091	156 ± 10.00	3.60 ± 0.34	33.80 ± 2.80	7.85 ± 0.57
3140	209 ± 12.00	3.51 ± 0.28	73.40 ± 3.90	11.12 ± 0.61
3701	124.00 ± 9.00	3.55 ± 0.60	25.90 ± 1.10	8.83 ± 0.57
3997	154.00 ± 11.00	5.54 ± 0.43	38.50 ± 3.20	14.10 ± 0.59
4036	187.00 ± 11.00	4.32 ± 0.71	51.10 ± 2.10	11.06 ± 0.75
4107	166.00 ± 10.00	3.20 ± 0.29	41.80 ± 1.07	8.94 ± 0.53
4368	163.00 ± 10.00	3.19 ± 0.35	43.60 ± 2.30	8.53 ± 0.48
4622	229.00 ± 14.00	7.56 ± 0.28	46.80 ± 8.50	17.92 ± 0.51
6903	143.00 ± 10.00	4.22 ± 0.83	28.20 ± 3.80	10.67 ± 1.03
7244	132.00 ± 9.00	3.86 ± 0.70	29.10 ± 1.60	8.66 ± 0.48
7917	249 ± 14.00	8.46 ± 0.39	69.90 ± 11.80	20.18 ± 0.94
12391	172.00 ± 10.00	3.86 ± 0.54	43.60 ± 3.30	9.39 ± 0.46

List of galaxies from Martinsson et al. (2013a) and Martinsson et al. (2013b) used. The first three properties are the reported observational estimates from the above authors, and R_c gives our estimate from the radial K-band surface brightness profiles. Galaxies UGC 00448, UGC 01635, UGC 03091, UGC 04036, UGC 04368 and UGC 12391 have well measured values for both disk velocity dispersion and mass surface density estimates at R_c , and were used in the estimates of $\langle Q_M \rangle$ and $\langle Q \rangle$ at the critical radii given at the end of section (3).

- Martin C.L., Kennicutt R.C., 2001, ApJ, 555, 301
- Martinsson T. P. K., Verheijen M. A. W., Westfall K. B., Bershady M. A., Schechtman-Rook A., Andersen D. R., Swaters R. A., 2013a, A&A, 557, 130
- Martinsson T. P. K., Verheijen M. A. W., Westfall K. B., Bershady M. A., Andersen D. R., Swaters R. A., 2013b, A&A, 557, 131
- Milgrom M., 1983, ApJ, 270, 365
- Milgrom M., 1989, ApJ, 338, 121
- Sanders R. H., McGaugh S. S., 2002, ARA&A, 40, 263
- Scarpa R., Marconi G., Carraro G., Falomo R., Villanova S., 2011, A&A, 525, A148
- Schaye J., 2004, ApJ, 609, 667
- Swaters R. A., Sanders R. H., McGaugh S. S., 2010, ApJ, 718, 380
- Toomre A., 1964, ApJ, 139, 1217